

Path analysis with partial association measures

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Abstract. This paper discusses the use of partial association measures for carrying out path analyses on categorical data. The measures considered are essentially PRE (proportion of reduction in error of prediction) measures for nominal variables and concordance–discordance indices for ordinal ones. These measures provide a natural way to evaluate the strength of the path linking a non-measurable response variable to one of several categorical explanatory factors. Concerning the decomposition of raw association into direct and indirect effects, it is shown, however, that they do not share the properties of conventional path coefficients for measurable variables. Especially purely nominal association measures need to be interpreted with care. The scope of the partial measures for path analysis is illustrated through a study of the relationships between the educational styles experienced by swiss adolescents and their self-esteem.

Key words: Path analysis, Categorical variables, Multiway contingency tables, Nominal and ordinal partial association measures.

1. Introduction

Regression and path analysis are widely used in the social sciences. Their interest lies in their ability to synthesize the way independent variables influence a dependent one. Through regression, one can quantify the strength and test the presence of a path between independent and dependent variables. Path analysis on interval variables systematizes this approach by considering the estimation of the standardized recursive linear system associated to a specified causal model. Among other results, traditional path analysis provides a decomposition of gross linkages, as measured by the Pearson's linear correlation coefficient, into direct and indirect effects. The knowledge of these decompositions is, for instance, of special relevance for policy issues.

Regression and hence traditional path analysis, require measurable variables. This limits strongly their scope in the social sciences where the concepts analyzed are often of a qualitative, i.e. non-measurable, nature. Think of

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the social status, the geographical origin, the ideology, the kind of studies. The obvious interest for regression-like coefficients and path analysis for non-measurable variables motivated this paper. The approach considered is based on partial association measures.

The literature on categorical data analysis and most modern statistical packages offer a lot of tools for studying the relationships between non-measurable variables. Log-linear modeling allows to detect pertinent association pattern. Techniques like factorial correspondence analysis focus on the linkages between categories while others like logistic or Poisson regression try to explain the probability to fall in a given category. Nevertheless, they do not provide synthesized measures of the linkage between variables as, for instance, the linear correlation and regression coefficients do for metric variables. Such synthesized measures are given by raw and partial association indices. Unlike the universal acceptance of correlation and regression coefficients as measures of linkage between interval variables there is however no such agreement for noninterval variables. The paper surveys the various alternatives which may be considered and discusses their relevance for path analysis.

Section 2 recalls the main features of path analysis for measurable variables. Section 3 surveys the main raw association indices for nominal and ordinal variables and Section 4 introduces partial association measures. General considerations on the use of partial association indices in path analysis are provided in Section 5. Finally, Section 6 presents a case study. It reconsiders some path analyses from Kellerhals *et al.* (1992a) who studied the relationships between the educational styles experienced by swiss adolescents and their self-esteem.

2. Classical path analysis

Before dealing with qualitative variables, we recall here the main features of classical path analysis, i.e. path analysis on measurable variables as introduced by Wright (1934) and described, for instance, in Blalock (1971) and Asher (1983).

The first step in path analysis is to specify a causal model, i.e. the recursive links between the dependent variable y and its explaining factors x_1, x_2, \dots, x_p . Figure 1 depicts such a causal structure for $p = 3$. Variables are usually assumed to be standardized, i.e. centered and having unit variance. Assuming linear relations, a causal, or recursive, model can then be written as

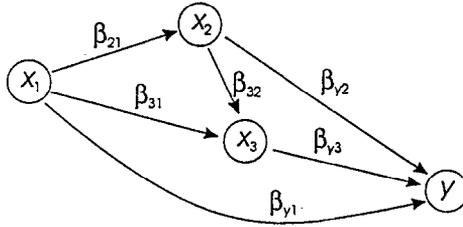


Fig. 1. A causal model.

$$\begin{aligned}
 x_2 &= \beta_{21}x_1 + u_2 \\
 x_3 &= \beta_{31}x_1 + \beta_{32}x_2 + u_3 \\
 &\vdots \\
 x_p &= \beta_{p1}x_1 + \dots + \beta_{p(p-1)}x_{p-1} + u_p \\
 y &= \beta_{y1}x_1 + \dots + \beta_{y(p-1)}x_{p-1} + \beta_{yp}x_p + u_y
 \end{aligned}$$

and the main concern of path analysis is the quantification of the path coefficients β_{ij} . With the classical assumptions on the error vectors u_i , i.e. $E(u_i) = 0$, $E(u_i u_i') = \sigma_i^2 I$ and $E(u_i u_j') = 0$, these path coefficients are efficiently estimated by the least squares estimators.

The estimate $\hat{\beta}_{st}$ measures the *direct influence* of variable t upon variable s . The value of the *indirect influence* $\beta_{t \rightarrow s}$ along a path of length $m + 1 > 1$ from t to s is derived from the path coefficients by the following chain rule:

$$b_{t \rightarrow s} = \hat{\beta}_{s i_1} \hat{\beta}_{i_1 i_2} \dots \hat{\beta}_{i_{m-1} i_m} \hat{\beta}_{i_m t}$$

There is indeed a unique path of length 1 ($m = 0$) and its value is $b_{t \rightarrow s} = \hat{\beta}_{st}$.

The *total influence* b_{ts}^{tot} of t upon s is then

$$b_{ts}^{tot} = \sum_{\mathcal{P}_{ts}} b_{t \rightarrow s} \tag{1}$$

where \mathcal{P}_{ts} is the set of paths from t to s . By excluding the direct influence, we get the total *indirect influence* b_{ts}^{ind}

$$b_{ts}^{ind} = \sum_{\mathcal{P}_{ts}^*} b_{t \rightarrow s} \tag{2}$$

where \mathcal{P}_{ts}^* is the set of paths from t to s of length greater than 1. We have $b_{ts}^{tot} = b_{ts}^{ind} + \hat{\beta}_{st}$.

It is worth to recall here that the total influence b_{ts}^{tot} is not equal to the raw linear correlation r_{ts} . This is readily shown by expressing r_{ts} in terms of the strengths $b_{i \rightarrow j}$ of direct and indirect paths.

Let $\{b_{k \rightarrow t}^q, b_{k \rightarrow s}^q\}$ denote a pair of values associated to two non-crossing paths starting at k and ending respectively in t and s . In other words, the paths $b_{k \rightarrow t}^q$ and $b_{k \rightarrow s}^q$ have, except for their starting point k , no common node i_j . Assuming without loss of generality that $t < s$, the raw correlation r_{ts} can be expressed as follows in terms of path values

$$r_{ts} = \sum_{\mathcal{P}_{ts}} b_{t \rightarrow s} + \sum_{k < t} \sum_q b_{k \rightarrow t}^q b_{k \rightarrow s}^q, \quad t < s. \quad (3)$$

The first summation sign is on the set \mathcal{P}_{ts} of direct and indirect paths from t to s . The last one bears on all possible pairs q of non-crossing paths with the same starting point and the adequate ending nodes t and s .

The total direct and indirect influence corresponds to the first term of the right hand member in Eqn. (3). It differs then from r_{ts} by the second term. This difference is known as spurious association and reflects indeed the joint dependence of s and t on common antecedent variables k .

Besides the decomposition of the correlation into direct, indirect and common external links, path coefficients are also of interest for decomposing the variance of the dependent variable. This aspect is nevertheless of less interest for our comparison with path analysis for non-interval variables and is not discussed here.

Pearson correlation and least-squares regression estimates are not applicable to non-measurable variables. In order to extend path analysis to such variables, we have thus to consider adequate alternatives for measuring the raw association on the one hand, and evaluate the strengths of the paths on the other one. Let us first investigate the first point by recalling the foundations of the main association measures.

3. Association measures for qualitative variables

This section describes shortly the main association measures for non-interval variables. We present first purely nominal measures, and, then, the ordinal ones. Obviously, the first can only measure the strength of the association, while the latter allow in addition to distinguish between positive and negative linkages.

The joint distribution of two categorical variables A and B is characterized by a simple two-way contingency table. Let A be the row variable with a set I of l categories, and B the column variable with a set J of c categories. We denote by p_{ij} the joint probability $P(A = i, B = j)$, by p_{i+} the probability of being in row i , i.e. $p_{i+} = P(A = i)$, and by p_{+j} the probability of being in

column j , i.e. $p_{+j} = P(B = j)$. These, indeed, define the theoretical distribution.

For estimation purposes, we consider the following sampled quantities: n_{ij} , the number of observations falling into the cell $(i; j)$, $n_{i+} = \sum_{j \in J} n_{ij}$ the total of observations in row i , $n_{+j} = \sum_{i \in I} n_{ij}$, the total in column j , and n the grand total of observations, i.e. $n = \sum_i \sum_j n_{ij}$.

3.1. Nominal measures

Purely nominal measures depend only on the matching of categories. They are independent of the order in which the categories are listed. Ideally, they are expected to take values between 0 (independence) and 1 (perfect association).

There are broadly two classes of measures; those based upon the Pearson Chi-Square Statistic, and those based upon the PRE (proportional reduction in error) interpretation.

3.1.1. Measures based upon the Chi-Square Statistic

Various normalized forms of the Pearson Chi-Square distance to independence $(\sum_i \sum_j (p_{ij} - p_{i+} p_{+j})^2 / p_{i+} p_{+j})$ have been proposed as association measures. These include the Pearson's Phi, the contingency coefficient (Pearson, 1904), Tschuprow's (1918) t and the Cramer's (1946) V . The latter is certainly the most widely used. Its sample estimate reads

$$\hat{V} = \sqrt{\frac{X^2}{n \min\{l - 1, c - 1\}}}$$

where $X^2 = n \sum_i \sum_j (n_{ij} - n_{i+} n_{+j} / n)^2 / n_{i+} n_{+j}$ is the Pearson Chi-Square Statistic, l the number of rows and c the number of columns. It is the only measure among those mentioned which is applicable to table of any size and can reach its bounds 0 and 1 whatever this size.

3.1.2. PRE measures

For predictive purposes, the measures derived from the Pearson Chi-Square are not satisfactory. Indeed, they do not strictly measure the association, but rather the distance to independence. Following Goodman & Kruskal (1954) an alternative approach has been considered which focuses on the predictive power of one variable for predicting the category of the other one. Broadly, this approach consists in measuring the proportion of the reduction in the error of prediction which results when the information on the first variable is used for predicting the second. This principle shares in some sense the

philosophy of regression and path analysis. Indeed, it presupposes, like regression and path analysis, a causal order between the variables, i.e. the distinction between dependent and independent variables.

Consider first the purely nominal case. Letting A be the dependent variable and B the independent one, the three main measures are:

$$\lambda_{AB} = \frac{\sum_j p_{mj} - p_{m+}}{1 - p_{m+}} \quad (4)$$

$$\tau_{AB} = \frac{\sum_i \sum_j \frac{p_{ij}^2}{p_{+j}} - \sum_i p_{i+}^2}{1 - \sum_i p_{i+}^2} \quad (5)$$

$$u_{AB} = \frac{\sum_i \sum_j p_{ij} \log_2 \left(\frac{p_{i+} p_{+j}}{p_{ij}} \right)}{\sum_i p_{i+} \log_2 p_{i+}} \quad (6)$$

where p_{mj} and p_{m+} are respectively the maximum in column j and the maximum among the row totals.

The first two are due to Goodman & Kruskal (1954). The first one, λ , presupposes a deterministic prediction rule while the second, τ , is based on a stochastic prediction rule. The third is Theil's uncertainty coefficient (Theil, 1970) based on Shannon's (1948) measure of entropy. It measures the gain in entropy, i.e. the gain in the average mass of information required to determine the correct category.

Sample estimates of the above measures are obtained by replacing the probabilities p_{ij} , p_{i+} and p_{+j} with the sample frequencies n_{ij}/n , n_{i+}/n and n_{+j}/n .

The exact distribution of these estimates is usually unknown. Statistical inference may nevertheless be done using asymptotic results. Formulas for computing asymptotic variances are given in Appendix. Note that the formula for the asymptotic variance of λ does not take into account the possibility for the maximum in each column to change row. It is therefore not valid in the neighborhood of points where such changes occur and, hence, especially unreliable near independence.

3.2. Ordinal measures

Concordance and discordance are the central notions of ordinal measures of association. Indeed, these measures catch the difference between the chances to find a pair of observations with concordant ranking on the two ordinal variables considered and the chances to find a discordant pair. The measures

take their value between -1 and 1 . A positive value means that a concordance is more likely than a discordance, while a negative value means that a discordance is more likely. Thus a value of 1 corresponds to a perfect positive association, and -1 to a perfect negative association.

The most popular measures, among which Kendall's τ_a (Kendall, 1938) and τ_b (Kendall, 1945), Stuart's (1953) τ_c , Goodman & Kruskal's (1954) γ , Somers's (1962) d , and Wilson's (1974) e , differ mainly in the way they account for ties. The first one, τ_a , can simply be expressed as the difference between the probability π^c of a concordant pair and that, π^d , of a discordant one

$$\tau_a = \pi^c - \pi^d.$$

The others are standardized forms of τ_a which, unlike τ_a , can reach their bounds -1 and 1 for contingency tables. Let π^t_A , π^t_B and π^t_{AB} denote respectively the probabilities of a pair with a tie on the dependent variable A only, on the independent variable B only, and on both variables A and B . The standardized measures read

$$\gamma = \frac{\pi^c - \pi^d}{\pi^c + \pi^d} \tag{7}$$

$$d_{AB} = \frac{\pi^c - \pi^d}{\pi^c + \pi^d + \pi^t_A} \tag{8}$$

$$e = \frac{\pi^c - \pi^d}{\pi^c + \pi^d + \pi^t_B + \pi^t_A} \tag{9}$$

$$\tau_b = \frac{\pi^c - \pi^d}{\sqrt{(\pi^c + \pi^d + \pi^t_A)(\pi^c + \pi^d + \pi^t_B)}} \tag{10}$$

$$\tau_c = \frac{\pi^c - \pi^d}{1 - 1/\min\{l, c\}}. \tag{11}$$

Alternative forms can easily be derived using for instance the following equalities

$$\begin{aligned} \pi^c + \pi^d + \pi^t_A &= 1 - \pi^t_B - \pi^t_{AB} \\ &= 1 - \sum_j p^2_{+j}. \end{aligned} \tag{12}$$

These measures refer indeed to different notions of perfect association. Obviously, the smaller the denominator, the weaker the notion is. The Goodman & Kruskal's γ , for instance, takes only account of the pairs without any tie. Thus, for $\pi^d = 0$, we get a perfect positive association even for very

small (but non-zero) values of π^c . It is also worth mentioning that, for non-square tables with no empty rows or columns, π_b and e cannot reach their bounds, and Somers's d can reach them only in the case where the dependent variable has at least as many categories than the independent one.

Among the above measures, the asymmetrical Somers's d is the best suited for predictive purposes and, hence, for a path-analysis framework. Indeed, it measures the probabilities among the pairs for which a significant information, i.e. a strict ranking, is available for the independent variable B .

Sample estimates of the ordinal measures are obtained by replacing the probabilities π^c , π^d , π_A^t , π_B^t and π_{AB}^t with the sample frequencies C/T , D/T , T^A/T , T^B/T and T^{AB}/T , where $T = n(n - 1)/2$ is the total number of pairs, C the number of concordant pairs, D the number of discordant pairs, T^A , T^B and T^{AB} the number of pairs with ties respectively only on A , only on B , and on both A and B .

Asymptotic variances of these estimates can be found in the Appendix.

4. Partial association measures

The nominal and ordinal measures of association discussed in Section 3 provide information on the raw association between variables. As soon as we are in presence of more than two variables, we need measures able to capture the direct link between two variables, i.e. the one which is not attributable to intermediate variables or common antecedents. One way to get such indicators is to measure the association by controlling for the states of the antecedents. A few such measures have been proposed in the literature for categorical variables. They are known as partial association measures. Goodman & Kruskal (1954) have discussed partial measures based on their λ_{AB} , and Davis (1967) has proposed a partial coefficient for Goodman and Kruskal's γ . See Quade (1974) for a more general discussion on partial correlation for metric and ordinal data.

For categorical variables, a partial coefficient is a weighted average of conditional association indices computed for each state of a third variable E . Assume E has a set K of q categories, and let $\theta_{AB|k}$ denote a generic association index measuring the association between A and B for a given state k of E . The partial measure is then

$$\theta_{AB|E} = \sum_{k \in K} \omega_k \theta_{AB|k}, \quad (13)$$

where the weights ω_k are non-negative and sum up to one.

This definition extends in a straightforward manner to the case of more

than three variables. Indeed, one has then simply to consider E as the set of all combinations of the categories of the variables which one wants to control for.

We have to precise how to chose the weights. A first approach, which corresponds to the first alternative considered by Goodman & Kruskal (1954), is to take the probabilities p_{++k} of each state k of the conditional variable E . A more satisfactory way is to base the weights on the denominator in the definition of the conditional indices.

For PRE nominal measures, this denominator is $p(\text{error}|k)$ the probability, when we are in state k of E , to make a prediction error on A in the absence of information on B . We suggest then to weight the conditional indices according to the joint probability of being in state k of E and making a prediction error, i.e. $p_{++k}p(\text{error}|k)$.

The weights are then of the following form

$$\omega_k = \frac{p_{++k}p(\text{error}|k)}{\sum_{k'} p_{++k'}p(\text{error}|k')} \tag{14}$$

For Goodman & Kruskal's lambda and tau, and Theil's u , these weights are respectively

$$\omega_k^\lambda = \frac{p_{++k} - p_{m+k}}{1 - \sum_{k'} p_{m+k'}} \tag{15}$$

$$\omega_k^\tau = \frac{p_{++k} - \sum_i p_{i+k}^2/p_{++k}}{1 - \sum_{k'} \sum_i p_{i+k'}^2/p_{++k'}} \tag{16}$$

$$\omega_k^u = \frac{\sum_i p_{i+k} \log_2 p_{i+k}/p_{++k}}{\sum_{k'} \sum_i p_{i+k'} \log_2 p_{i+k'}/p_{++k'}} \tag{17}$$

We obtain thus the following partial lambda $\lambda_{AB|E}$, partial (nominal) tau $\tau_{AB|E}$, and partial uncertainty coefficient $u_{AB|E}$

$$\lambda_{AB|E} = \frac{\sum_k (\sum_j p_{mjk} - p_{m+k})}{1 - \sum_k p_{m+k}} \tag{18}$$

$$\tau_{AB|E} = \frac{\sum_k \left(\sum_i \sum_j \frac{p_{ijk}^2}{p_{+jk}} - \sum_i \frac{p_{i+k}^2}{p_{++k}} \right)}{1 - \sum_k \sum_i \frac{p_{i+k}^2}{p_{++k}}} \tag{19}$$

$$u_{AB|E} = \frac{\sum_k \sum_i \sum_j p_{ijk} \log_2 \left(\frac{p_{i+k} p_{+jk}}{p_{ijk} p_{++k}} \right)}{\sum_k \sum_i p_{i+k} \log_2 \frac{p_{i+k}}{p_{++k}}}. \quad (20)$$

Note that the index (18) is precisely Goodman & Kruskal's second alternative.

A similar approach has been advocated by Quade (1974) for those ordinal association measures which can be expressed in the form

$$\theta = \frac{\pi^c - \pi^d}{\pi^r}, \quad (21)$$

where π^r denotes the probability to get a relevant pair. For Somers's d_{AB} , for example, π^r is given by $\pi^c + \pi^d + \pi^t_A$. Partial ordinal measures are thus obtained by averaging the conditional indices according to the probabilities π^r_k of getting a relevant pair tied on the k th state of E . It is worth mentioning that the π^r_k 's are joint probabilities, i.e.

$$\pi^r_k = p^2_{++k} \pi^{r|k}, \quad (22)$$

where p^2_{++k} is the probability to get a pair with both observations in the k th subtable, and $\pi^{r|k}$ the conditional probability of a relevant pair among those tied on the state k of E .

Resulting partial measures for γ , τ_b and Somers's d , for example, are then

$$\gamma_{AB|E} = \frac{\sum_k (\pi^c_k - \pi^d_k)}{\sum_k (\pi^c_k + \pi^d_k)} \quad (23)$$

$$d_{AB|E} = \frac{\sum_k (\pi^c_k - \pi^d_k)}{\sum_k (\pi^c_k + \pi^d_k + \pi^t_{Ak})} \quad (24)$$

$$\tau^b_{AB|E} = \frac{\sum_k (\pi^c_k - \pi^d_k)}{\sum_k (\pi^c_k + \pi^d_k + \pi^t_{Ak})^{1/2} (\pi^c_k + \pi^d_k + \pi^t_{Bk})^{1/2}} \quad (25)$$

where π^t_{Ak} denotes the probability to get a pair tied on A and the k th state of E but not on B .

Let π^c_k , π^d_k and π^t_{Ak} denote the probabilities to get, among the pairs tied on state k of E , a pair which is respectively concordant on A and B , discordant on A and B , or untied on B . The probability to get a pair tied on state k is p^2_{++k} . Likewise, the probability to get a pair tied on state i of A and k of E is p^2_{i+k} . We have thus

$$\pi^c_k + \pi^d_k + \pi^t_{Ak} = p^2_{++k} - \sum p^2_{+jk} \quad (26)$$

$$\pi_k^c + \pi_k^d + \pi_{Bk}^e = p_{++k}^2 - \sum_i p_{i+k}^2 \tag{27}$$

$$\pi_k^c + \pi_k^d = p_{++k}^2 - \sum_i p_{i+k}^2 - \sum_j p_{+jk}^2 + \sum_j \sum_i p_{ijk}^2 \tag{28}$$

These equalities are indeed of interest for computational purposes.

Weights based on quantities involved in the construction of the conditional measures are, unlike the p_{++k} 's, coherent with the philosophy of the underlying measures. Furthermore, they provide partial measures which can easily be expressed as ratios of summations. This is important, since, for those indices which are derivable with respect to the p_{ijk} 's, i.e. all those presented but $\lambda_{AB|E}$, it allows to derive asymptotic variances for their estimates by a straight application of the δ -rule. The asymptotic variances are given in the Appendix.

5. Partial associations as path coefficients

Now that we have introduced the partial association measures, let us discuss their relevance for path analysis.

A partial association index provides an evaluation of the strength of the associated connection. But can we use and interpret these measures like path coefficients for measurable variables? This requires unfortunately some cautions.

The strength of a connection as measured by a partial association measure may be viewed as the direct effect of the independent variable upon the response variable. Likewise, the product of the partial measures along a path of length greater than two going from one variable to another one may also be interpreted as the intensity of the effect which works along this path. In that sense it represents an indirect effect.

However, the association measures are usually not invariant with the breakdown of categories. For instance, consider the two following contingency tables:

	B_1	B_2	B_3		B_1, B_2	B_3	
A_1	0.1	0	0.15	A_1, A_2	0.25	0.25	
A_2	0	0.15	0.1	A_3	0.25	0.25	(29)
A_3	0.15	0.1	0.25				

The second is obtained from the first one by aggregating the first two rows and columns. In this case, aggregation of categories weakens the association.

This is what one could expect from the information lost by aggregation. However, as shown in the next example, aggregation may sometimes reinforce the association.

	B_1	B_2	B_3		B_1, B_2	B_3	
A_1	0.1	0.3	0	A_1, A_2	0.7	0	
A_2	0.2	0.1	0	A_3	0	0.3	(30)
A_3	0	0	0.3				

The point is that the associations and partial associations are conditional to the retained aggregation level. This should be kept in mind when comparing and combining different direct and indirect effects between two variables. An indirect effect may be relatively strong due to the high breakdown of the categories of an intermediate variable, while another one may be relatively low because the intermediate variable has only two aggregated categories.

Consider now the decomposition of the raw association between two variables into direct and indirect influences transmitted along the various paths which lead from one variable to the other one. We have shown in Section 2 that the raw correlation can be decomposed in terms of traditional path coefficients (Eqn. (3)). Here, there is no such formal relationship between partial association measures and their corresponding raw measure. Note that, even for measurable variables, the relation (3) does not hold if we consider partial correlations instead of the (regression) path coefficients.

Partial measures based on unsigned association indices, as for instance the nominal measures λ , τ and u , are especially inappropriate to this decomposition purpose. Look for example at the following contingency table

	B_1	B_2	
A_1	0.25	0.25	
A_2	0.25	0.25	(31)

and its decomposition according to a third variable E

E_1	B_1	B_2	E_2	B_1	B_2	
A_1	0.25	0	A_1	0	0.25	
A_2	0	0.25	A_2	0.25	0	(32)

This decomposition is analogous to the following one according to the variable B

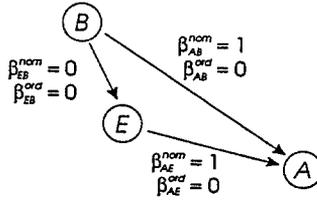


Fig. 2. Paths valued with nominal and ordinal partial measures.

B_1	E_1	E_2	B_2	E_1	E_2	(33)
A_1	0.25	0	A_1	0	0.25	
A_2	0	0.25	A_2	0.25	0	

The raw association between A and B is obviously null, and it is readily shown that the one between A and E , and between B and E are null too. The conditional associations are, nevertheless, perfect, and hence the partial nominal associations are equal to one. Thus, valuing the paths of Figure 2 with partial nominal association indices leads to a total influence of B upon A equal to one, which contrasts with the null raw association.

This problem does not arise with ordinal partial measures which take account, through the sign of the conditional associations, of the structural difference between the subtables. For our example, valuing the paths with ordinal partial association indices leads to a null total influence, which is coherent with the raw association.

Our experience confirms this point. Though there is no formal relation between partial and raw ordinal association measures, we have observed that raw ordinal associations are usually almost equal to the sum of the direct and indirect effects.

In traditional path analysis, the raw association, i.e. the raw correlation, is symmetrical while the path coefficients depend upon the response variable. By analogy for ordinal variables, we suggest then to use Kendall's τ_b , which is symmetric, for the raw association and Somers's d , which is asymmetric, for the path coefficients. Indeed, as we have already pointed out, the Somers's d 's are regression-like coefficients in the sense that they depend upon the choice of the response variable: $d_{AB} \neq d_{BA}$. Furthermore, it is readily shown that we have

$$d_{AB}d_{BA} = \tau_b^2. \tag{34}$$

This relation is similar to the one between single regression least-squares coefficients and the square of Pearson's correlation: $\hat{\beta}_x \hat{\beta}_y = r_{xy}^2$, where $\hat{\beta}_x$ and

$\hat{\beta}_y$ are the OLS slope estimates of the regression respectively of y upon x and x upon y .

6. A case study

For illustrative purposes, this section reconsiders a path analysis carried out by Kellerhals *et al.* (1992a). The goal of the study was to analyze the relationships between the educational environment experienced by Swiss adolescents and their self-esteem.

The data considered for describing the educational environment are derived from those collected by a research team of the University of Geneva (Kellerhals & Montandon, 1991). This team conducted a study amongst a random sample of families with a boy or girl aged 13. The parents were interviewed and the children provided data through a self-report questionnaire. The study aimed at characterizing various educational patterns and at determining their relationships with the familial environment, i.e. mainly the social status of the family (as approximated by that of the father) and the nature of the interaction of the family. The latter is defined by combining axes of internal cohesion and external integration. There are four ideal-type of family-interaction patterns. *Parallel* families are characterized by closure and autonomy, *Shelter* families are characterized by closure and fusion, *Companionship* families are at the same type highly cohesive and open and *Associative* families are characterized by openness and autonomy.

Concerning the educational practices, three main styles emerged from a cluster analysis of 22 indicators referring to four aspects of the educational process: educational objectives, techniques of influence, role distribution among parents and coordination with other educational agents. The first style, called 'statutory', is characterized by a rigorous division of labor among spouses, low communication between parents and children, high accommodation, prevalence of control as a method of education and hostility to external educational influence. The second style, called 'maternalistic', differs from the precedent one by the important proximity observed between the mother and the children (high communication, many activities in common high empathy). Finally, the 'contractualistic' style is characterized by a high emphasis on self-regulation and sensibility, the use of relational methods, a relative lack of differentiation between paternal and maternal roles and a high permeability to external influence. See Kellerhals *et al.* (1992b) for more details.

The Self-Esteem index is described in Kellerhals *et al.* (1992a). It includes two dimensions: the self-efficacy of the child (number of actions that she or

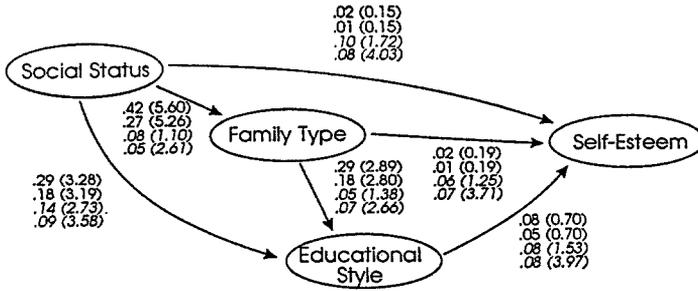


Fig. 3. Girls and Boys: Complete model. The path coefficients are given in the order $\hat{\gamma}_{AB|E}$, $\hat{d}_{AB|E}$, $\hat{\lambda}_{AB|E}$ and $\hat{\tau}_{AB|E}$. The nominal path coefficients are italicized. The t -values are given between brackets.

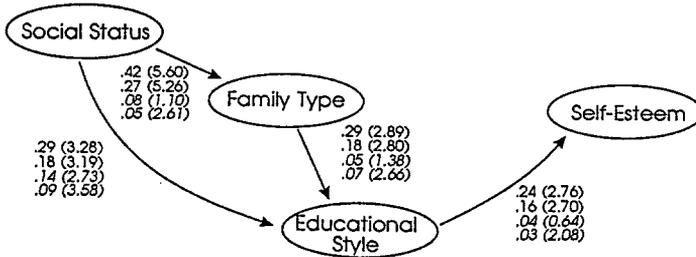


Fig. 4. Girls and Boys: Final model. The path coefficients are given in the order $\hat{\gamma}_{AB|E}$, $\hat{d}_{AB|E}$, $\hat{\lambda}_{AB|E}$ and $\hat{\tau}_{AB|E}$. The nominal path coefficients are italicized. The t -values are given between brackets.

he could do alone) on the one hand, and its self-worth (number of qualities, as compared with peers, that she or he claims) on the other hand. Childs rated above the mean on the two scales exhibit a good Self-Esteem.

The data set considered for our illustration concerns thus 249 thirteen years old girls and boys. The educational environment is described by means of three ordinal variables. These are the Social Status *SS* ('Workers or Employees', 'Junior Executives', 'Senior Executives', 'University-trained Professionals'), the Family Type *FT* ('Parallel or Shelter', 'Companionship', 'Association') and the Educational Style *ES* ('Statutory', 'Maternalistic', 'Contactualistic'). The Self-Esteem *SE* is represented by a three levels ('Low', 'Middle', 'High') variable.

Figure 3 exhibits the causal structure of the analyzed model. The paths are valued with partial γ 's and d 's. Though the variables are ordinal, we provide also the values of two nominal partial measures: the partial λ 's and

τ 's. The t -value, i.e. the ratio between the estimated coefficient and its standard error, is given next to each values. Broadly, a t -value larger than 2 indicates that the coefficient is statistically significant.

Note that the values of the partial γ 's differ from those given in Kellerhals *et al.* (1992a). This is because the latter were computed using Goodman & Kruskal's first method, i.e. by averaging the conditional γ 's according to the size of the subsamples. The ones given here were obtained with the second alternative, i.e. formula (23), which gives less importance to the conditional tables with low frequencies.

Let us look at the ordinal partial coefficients. The partial γ 's and d 's provide quite similar results, though, as expected from their definitions, the d 's are smaller than the γ 's. The paths linking the three environmental variables are clearly significant. On the other hand, none of the paths linking these variables to Self-Esteem is significant. As can be shown from Table I, the raw association between the Educational Style and Self-Esteem is nevertheless significant. We face thus here a phenomenon similar to collinearity. The remedy is to discard some insignificant paths. Figure 4 shows the model obtained after elimination of the paths linking directly Social Status and Family Type to Self-Esteem. All remaining ordinal path coefficients are now significant.

The structure of this final model agrees with the conclusions drawn in Kellerhals *et al.* (1992a). It agrees also with the association pattern corresponding to the best model obtained with a hierarchical log-linear analysis. It is worth mentioning, however, that hierarchical log-linear analysis does not take account of the ordinality of the variables. Furthermore, it says nothing about the direction of causality.

Curiously, nominal partial measures do not provide the same conclusions. Partial λ 's seem to be systematically insignificant while partial τ 's look all as significant, even for the complete model. Note, however, that the nominal measures themselves have very low values. We are thus very close to independence where the nominal measures have a non normal behavior. Indeed, since they cannot take negative values, nominal measures have obviously a skewed distribution near zero. Furthermore, a simulation study in Olszak & Ritschard (1995) showed that, for nominal measures close to zero, the estimated asymptotic standard error overestimates significantly the true standard deviation. Thus, in our example, no definite conclusion can be drawn about the nominal measures from their t -values. The simulation showed also that the λ 's have, near independence, a much higher standard deviation than the τ 's. This may explain the differences observed here in the t -values for the λ 's and τ 's.

Let us now consider the decomposition of the raw association. Between

Table I. Raw associations

	Social status	Family type	Educational style
<i>Girls and Boys (249)</i>			
$\hat{\gamma}$			
Family type	0.42 (5.60)		
Educational style	0.39 (5.08)	0.42 (5.39)	
Self-esteem	0.15 (1.64)	0.16 (1.76)	0.24 (2.76)
$\hat{\tau}_b$			
Family type	0.27 (5.30)		
Educational style	0.26 (4.81)	0.28 (5.11)	
Self-esteem	0.10 (1.63)	0.10 (1.75)	0.16 (2.71)
<i>Girls (129)</i>			
$\hat{\gamma}$			
Family type	0.40 (3.93)		
Educational style	0.41 (3.95)	0.39 (3.77)	
Self-esteem	0.19 (1.59)	0.13 (1.08)	0.17 (1.29)
$\hat{\tau}_b$			
Family type	0.26 (3.72)		
Educational style	0.27 (3.70)	0.26 (3.61)	
Self-esteem	0.12 (1.57)	0.08 (1.08)	0.11 (1.28)
<i>Boys (120)</i>			
$\hat{\gamma}$			
Family type	0.43 (3.98)		
Educational style	0.36 (3.22)	0.46 (3.91)	
Self-esteem	0.10 (0.78)	0.10 (1.36)	0.32 (2.72)
$\hat{\tau}_b$			
Family type	0.28 (3.77)		
Educational style	0.24 (3.08)	0.30 (3.67)	
Self-esteem	0.07 (0.77)	0.12 (1.34)	0.21 (2.64)

Values between brackets are *t*-values, i.e. $\hat{\gamma}/\hat{\sigma}_{\hat{\gamma}}$ and $\hat{\tau}_b/\hat{\sigma}_{\hat{\tau}_b}$.

Social Status and Educational Style, we have respectively $\gamma = 0.39$ and $\tau_b = 0.26$. Summing the two paths linking these same variables we get very close values (Table II). We can thus conclude that the association between Social Status and Educational Style is due for about 20–30% to the indirect link through the Family Type.

Summing the two indirect paths going from Social Status to Self-Esteem we get only a half of the raw association with the partial γ 's, and a third with the partial d 's. The raw association between the Social Status and the Self-Esteem is, however, not statistically significant. It makes then no sense to examine its decomposition.

Examining the links between the variables for girls only on the one hand, and for boys only on the other hand, is instructive. Figure 5 shows that the

Table II. Paths from Social Status to Educational Style

Path		Path valued with	
		Partial γ	Partial d
$SS \rightarrow FT \rightarrow ES$	=	0.12	0.05
$SS \rightarrow ES$	=	0.29	0.18
Total		0.41	0.23

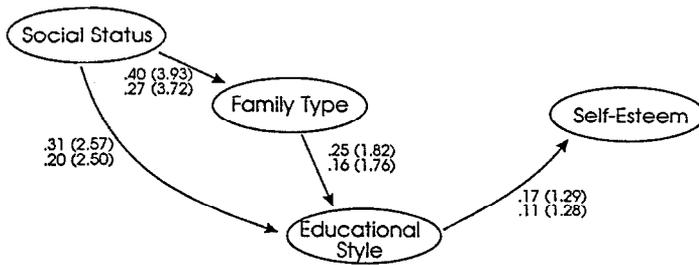


Fig. 5. Girls: Path coefficients are respectively $\hat{\gamma}_{AB|E}$ and \hat{d}_{ABE} . Next to each coefficient is the t -value.

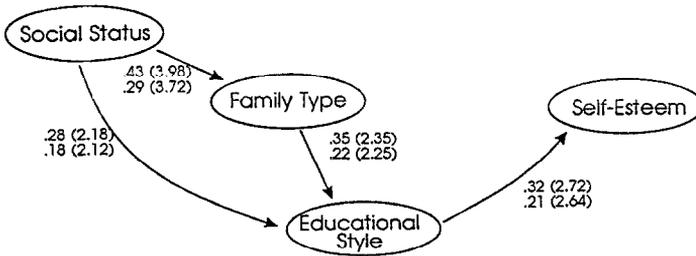


Fig. 6. Boys: Path coefficients are respectively $\hat{\gamma}_{AB|E}$ and $\hat{d}_{AB|E}$. Next to each coefficient is the t -value.

association pattern pointed out does not hold for girls. The path from Family Type to Educational Style and the one from Educational Style to Self-Esteem are not significant. This is also confirmed by the log-linear analysis which leads to the model with generating class ($SS * FT, SS * ES, SE$). When only boys are considered, however, we get the same final model (Figure 6).

7. Conclusion

It has been shown that partial association measures are useful tools for valuing and testing paths in a causal model between categorical variables. Nevertheless, their interpretation as path coefficients requires some caution. Purely nominal partial association measures are, for instance, badly suited for decomposing a raw association into direct and indirect effects. Though the decomposition formula which expresses the raw correlation in terms of classical regression-path coefficients has no equivalent for raw and partial association measures, experience shows that the sum of the direct and indirect paths, as measured with ordinal partial indices, is almost equal to the raw association.

Partial association measures provide synthesized information not available from more modern tools based on the fitting of parametric models. In order to benefit from the numerous advantages of the latter, a model in which path coefficients between categorical variables could be introduced as parameters remains to be developed. This is certainly an appealing challenge for the research on the analysis of categorical data.

Appendix: Asymptotic variances

We give hereafter the asymptotic variances of the various raw and partial association measures considered. The formulas were derived using the δ -rule.

Broadly, according to this δ -rule, the asymptotic variance of the estimate of a ratio of the form $\theta = \nu(\dots, p_{ij}, \dots) / \delta(\dots, p_{ij}, \dots)$ derivable with respect to the p_{ij} 's, is

$$\sigma_{\infty}^2(\hat{\theta}) = \frac{1}{n\delta^4} \sum_{i \in J} \sum_{j \in J} p_{ij} (\phi_{ij} - \bar{\phi})^2 \tag{35}$$

with $\phi_{ij} = \nu(\partial\delta/\partial p_{ij}) - \delta(\partial\nu/\partial p_{ij})$ and $\bar{\phi} = \sum_i \sum_j p_{ij} \phi_{ij}$.

The raw and partial λ 's rely on maxima and are therefore not derivable with respect to the p_{ij} 's or p_{ijk} 's at points where probability maxima move from one row to another. The δ -rule applies then only outside these points.

A1. *For estimates of raw measures*

The asymptotic variance of the estimate $\hat{\lambda}$ is

$$\sigma_{\infty}^2(\hat{\lambda}_{AB}) = \frac{(1 - \sum_{j \in J} p_{mj})(p_{m+} + \sum_{j \in J} p_{mj} - 2 \sum_{j \in J_{m+}} p_{mj})}{n(1 - p_{m+})^3} \quad (36)$$

where J_{m+} is the set of columns whose maximum is in the same row as the marginal maximum p_{m+} . See Goodman & Kruskal (1963). This formula does not take account of possible row changes of the maxima.

For the estimate $\hat{\tau}$ of Goodman & Kruskal's τ we have $\delta = 1 - \sum_i p_{i+}^2$ and

$$\phi_{ij} = 2p_{i+} \left(1 - \sum_{i'} \sum_{j'} \frac{p_{i'j'}}{p_{+j'}} \right) + \left(1 - \sum_{i'} p_{i'+}^2 \right) \left(2 \frac{p_{ij}}{p_{+j}} - \sum_{i' \in J} \left(\frac{p_{i'j}}{p_{+j}} \right)^2 \right). \quad (37)$$

For the asymptotic variance of the estimate \hat{u} of the uncertainty coefficient, we get the expression

$$\sigma_{\infty}^2(\hat{u}_{AB}) = \frac{\sum_i \sum_j p_{ij} (H(A) \log_2(p_{ij}/p_{+j}) - H(A|B) \log_2 p_{i+})^2}{nH(A)^4} \quad (38)$$

where $H(A)$ denotes the entropy of A , i.e. $H(A) = -\sum_i p_{i+} \log_2 p_{i+}$, and $H(A|B)$ the entropy of A conditional to B , i.e. $H(A|B) = -\sum_j \sum_i (p_{ij}/p_{+j}) \log_2(p_{ij}/p_{+j})$. The same formula has been derived by Agresti (1986).

For the ordinal measures $\hat{\tau}_a$, $\hat{\gamma}$, \hat{d}_{AB} and \hat{e} , we get

$$\sigma_{\infty}^2(\hat{\tau}_a) = \frac{4}{n} \left(\sum_i \sum_j p_{ij} (\pi_{ij}^c - \pi_{ij}^d)^2 - \tau_a^2 \right) \quad (39)$$

$$\sigma_{\infty}^2(\hat{\gamma}) = \frac{16}{n(\pi^c + \pi^d)^4} \sum_i \sum_j p_{ij} (\pi^c \pi_{ij}^d - \pi^d \pi_{ij}^c)^2 \quad (40)$$

$$\sigma_{\infty}^2(\hat{d}_{AB}) = \frac{4 \sum_i \sum_j p_{ij} ((\pi^c - \pi^d)(1 - p_{+j}) - (1 - \sum_{j'} p_{+j'}^2)(\pi_{ij}^c - \pi_{ij}^d))^2}{n(1 - \sum_j p_{+j}^2)^4} \quad (42)$$

$$\sigma_{\infty}^2(\hat{e}) = \frac{4 \sum_i \sum_j p_{ij} ((\pi^c - \pi^d)(1 - p_{ij}) - (1 - \sum_{i'} \sum_{j'} p_{i'j'}^2)(\pi_{ij}^c - \pi_{ij}^d))^2}{n(1 - \sum_i \sum_j p_{ij}^2)^4} \quad (42)$$

where π_{ij}^c and π_{ij}^d denote the probabilities to get an observation respectively concordant or discordant with those in cell (i, j) , i.e.

$$\pi_{ij}^c = \sum_{i' < i} \sum_{j' < j} p_{i'j'} + \sum_{i' > i} \sum_{j' > j} p_{i'j'} \quad (43)$$

$$\pi_{ij}^d = \sum_{i' < i, j' > j} p_{i'j'} + \sum_{i' > i, j' < j} p_{i'j'}. \quad (44)$$

The asymptotic variance of $\hat{\tau}_b$ is given by (35) with

$$\delta = (1 - \sum_i p_{i+}^2)^{1/2} (1 - \sum_j p_{+j}^2)^{1/2}$$

and

$$\begin{aligned} \phi_{ij} = & 2(\pi_{ij}^c - \pi_{ij}^d) \sqrt{(1 - \sum_{i'} p_{i'+}^2)(1 - \sum_{j'} p_{+j'}^2)} + \\ & + \tau_b(p_{i+}(1 - \sum_{j'} p_{+j'}^2) + p_{+j}(1 - \sum_{i'} p_{i'+}^2)). \end{aligned} \quad (45)$$

Since l and c are sample invariant, the asymptotic variance of $\hat{\tau}_c$ follows directly from that of $\hat{\tau}_a$

$$\sigma_{\infty}^2(\hat{\tau}_c) = \left(\frac{\min\{l, c\}}{1 - \min\{l, c\}} \right)^2 \sigma_{\infty}^2(\hat{\tau}_a). \quad (46)$$

A2. For estimates of partial association

Goodman & Kruskal (1963; 333) give the asymptotic variance of $\hat{\lambda}_{AB|E}$

$$\sigma_{\infty}^2(\hat{\lambda}_{AB|E}) = \frac{(1 - \sum_k \sum_j p_{mjk})(\sum_k \sum_j p_{mjk} + \sum_k p_{m+k} - 2\sum_k \sum_{j \in J_{m+k}} p_{mjk})}{(1 - \sum_k p_{m+k})^3}. \quad (47)$$

For the other partial indices, the δ -rule provides asymptotic variances in the form

$$\sigma_{\infty}^2(\hat{\theta}_{AB|E}) = \frac{1}{n\delta^4} \sum_k \sum_j \sum_k p_{ijk} (\phi_{ijk} - \bar{\phi})^2 \quad (48)$$

where $\bar{\phi}$ is $\sum_k \sum_j \sum_i p_{ijk} \phi_{ijk}$. We give hereafter the quantities δ and ϕ_{ijk} for the nominal measures $\hat{\tau}_{AB|E}$ and $\hat{u}_{AB|E}$, and the ordinal $\hat{\tau}_{AB|E}^b$.

For $\hat{\tau}_{AB|E}$, we have

$$\delta = 1 - \sum_k \sum_i \frac{p_{i+k}^2}{p_{++k}} \quad (49)$$

$$\delta_{ijk} = \left(\frac{-2p_{i+k}}{p_{++k}} + \frac{1}{p_{++k}^2} \sum_{i'} p_{i'+k}^2 \right) \left(1 - \sum_{k'} \sum_{j'} \sum_{i'} \frac{p_{i'j'k'}^2}{p_{+j'k'}} \right) -$$

$$- \delta \left(\frac{1}{p_{++k}^2} \sum_i p_{i'jk}^2 - \frac{2p_{ijk}}{p_{++k}} \right). \quad (50)$$

For $\hat{u}_{AB|E}$, we have

$$\delta = \sum_k \sum_i p_{i+k} \log_2 \frac{p_{i+k}}{p_{++k}} \quad (51)$$

$$\begin{aligned} \phi_{ijk} &= \log_2 \left(\frac{p_{i+k}}{p_{++k}} \right) \sum_{k'} \sum_{i'} \sum_{j'} p_{i'j'k'} \log_2 \left(\frac{p_{i'+k'} p_{+j'k'}}{p_{i'j'k'} p_{++k'}} \right) - \\ &\quad - \delta \log_2 \left(\frac{p_{i+k} p_{+jk}}{p_{ijk} p_{++k}} \right). \end{aligned} \quad (52)$$

For $\hat{r}_{AB|E}^b$, we have

$$\delta = \sum_k \left(\left(p_{++k}^2 - \sum_i p_{i+k}^2 \right) \left(p_{++k}^2 - \sum_j p_{+jk}^2 \right) \right)^{1/2} \quad (53)$$

$$\begin{aligned} \phi_{ijk} &= -2\delta (\pi_{ij,k}^c - \pi_{ij,k}^d) + \left(\sum_{k'} (\pi_{k'}^c - \pi_{k'}^d) \right) \cdot \\ &\quad \cdot \left(\frac{(p_{++k}^2 - \sum_{i'} p_{i'+k}^2)(p_{++k} - p_{+jk}) + (p_{++k}^2 - \sum_{j'} p_{+j'k}^2)(p_{++k} - p_{i+k})}{(p_{++k}^2 - \sum_{i'} p_{i'+k}^2)^{1/2} (p_{++k}^2 - \sum_{j'} p_{+j'k}^2)^{1/2}} \right) \end{aligned} \quad (54)$$

where $\pi_{ij,k}^c$ and $\pi_{ij,k}^d$ designate the probabilities to get a case which falls in category k of E and is respectively concordant or discordant according to the first two variables A and B with those in cell (i, j, k) , i.e.

$$\pi_{ij,k}^c = \sum_{i' < i} \sum_{j' < j} p_{i'j'k} + \sum_{i' > i} \sum_{j' > j} p_{i'j'k} \quad (55)$$

$$\pi_{ij,k}^d = \sum_{i' < i} \sum_{j' > j} p_{i'j'k} + \sum_{i' > i} \sum_{j' < j} p_{i'j'k}. \quad (56)$$

For the ordinal indices $\hat{\gamma}_{AB|E}$ and $\hat{d}_{AB|E}$ the expressions are a little simpler. Here is the final form of their asymptotic variances in terms of the probabilities π_k^c of a concordant pair and π_k^d of a discordant pair among those tied on state k of E

$$\sigma_{\infty}^2(\hat{\gamma}_{AB|E}) = \frac{16}{n(\sum_k (\pi_k^c + \pi_k^d))^4} \sum_k \sum_j \sum_i p_{ijk} \left(\pi_{ij,k}^d \sum_{k'} \pi_{k'}^c - \pi_{ij,k}^c \sum_{k'} \pi_{k'}^d \right)^2 \quad (57)$$

$$\sigma_{\hat{d}_{AB|E}}^2 = \frac{4}{n} \left(\sum_k (\pi_k^c + \pi_k^d + \pi_{Ak}^t) \right)^{-4} \sum_k \sum_j \sum_i p_{ijk} \times \\ \times \left((p_{++k} - p_{+jk}) \sum_{k'} (\pi_{k'}^c - \pi_{k'}^d) - (\pi_{ij,k}^c - \pi_{ij,k}^d) \sum_{k'} (\pi_{k'}^c + \pi_{k'}^d + \pi_{Ak'}^t) \right)^2. \quad (58)$$

The sample validity of these asymptotic variances has been assessed in Olszak & Ritschard (1993) by means of a simulation study.

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