# Supervised and non-supervised analysis of the diversity of categorical sequences 

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## Outline

## (1) Objectives

(2) Pairwise dissimilarities between sequences
(3) Unsupervised clustering

4 Supervised Classification
(5) Conclusion

## Outline

## (1) Objectives

(2) Pairwise dissimilarities between sequences
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## Aim of the presentation

- Using Swiss cohabitational and occupational data from the SHP biographical survey
- Demonstrate the results that can be extracted from state sequences
- Unsupervised learning
- Clustering
- Representative sequences

Supervised learning

- Discrepancy analysis (ANOVA)
- Regression trees of state sequence data


## State sequences: examples

- Cohabitational state sequences (from SHP) $2 \mathrm{P}=$ with 2 parents, $\mathrm{U}=$ with partner, $\mathrm{C}=$ with child, $\mathrm{A}=$ alone, $\ldots$
Sequence
$12 \mathrm{P}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}-\mathrm{U}$
2 2P-2P-U-U-U-U-U-U-UC-UC-UC-UC-UC-UC-UC-UC-UC-UC-UC-UC-UC-UC-UC-UC-UC-UC
$32 \mathrm{P}-\mathrm{A}-\mathrm{A}-\mathrm{A}-\mathrm{A}-\mathrm{A}-\mathrm{U}-\mathrm{U}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}$
4 2P-2P-2P-2P-2P-2P-2P-2P-U-UC-UC-UC-UC-UC-UC-UC-UC-UC-UC-UC-UC-UC-UC-UC-UC-UC


## - Compact representation

## Sequence

1 (2P, 1)-(U, 25)
$2(2 P, 2)-(U, 6)-(U C, 18)$
3 (2P, 1)-(A,5)-(U, 2)-(UC, 18)
$4(2 \mathrm{P}, 8)-(\mathrm{U}, 1)-(\mathrm{UC}, 17)$


## State sequences: examples

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$42 \mathrm{P}-2 \mathrm{P}-2 \mathrm{P}-2 \mathrm{P}-2 \mathrm{P}-2 \mathrm{P}-2 \mathrm{P}-2 \mathrm{P}-\mathrm{U}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}-\mathrm{UC}$


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```



## Typical questions in social sciences

- In the field of Life course analysis
- Are there standard of lives, ideal-types?
- What are those standards, those ideal-tyes?
- How are those standards linked to covariates (sex, birth cohort, ...)?
- Can answer to these questions by seeking typologies and studying groupings in terms of covariates


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## Prerequisite: pairwise dissimilarities

- Common point between all addressed methods:
- ... all are based on pairwise dissimilarities between sequences


## Outline

## (1) Objectives

(2) Pairwise dissimilarities between sequences
(3) Unsupervised clustering
(4) Supervised Classification
(c) Conclusion

## Dissimilarity measure - 1

Based on count of matching attributes

- Measures Based on count of matching attributes $A(x, y)$ (proximity)

$$
d(x, y)=A(x, x)+A(y, y)-2 A(x, y)
$$

available in TraMineR

- $\operatorname{LCP} A(x, y)=$ length of longest common prefix
- RLCP $A(x, y)=$ length of longest common suffix
- LCS $A(x, y)=$ length of longest common subsequence
- HAM simple $A(x, y)=$ half of number of matching elements


## Dissimilarity measure - 2

## Edit distances

- Edit distance: (minimal) cost of transforming $x$ into $y$ available in R
- OM Optimal matching of state séquences (Levenshtein, 1966)
- indel cost (insertion/deletion)
- pairwise substitution costs
- Generalized HAM, Hamming = OM without indel
- DHD, Dynamic Hamming Distance, position-varying substitution cost (Lesnard, 2006)


## Discrepancy of a set of sequences

- From a dissimilarity matrix, we can define the discrepancy of a set of sequences
- Sum of squares $S S$ can be expressed in terms of pairwise distances

$$
\begin{aligned}
S C & =\sum_{i=1}^{n}\left(y_{i}-\bar{y}\right)^{2}=\frac{1}{n} \sum_{i=1}^{n} \sum_{j=i+1}^{n}\left(y_{i}-y_{j}\right)^{2} \\
& =\frac{1}{n} \sum_{i=1}^{n} \sum_{j=i+1}^{n} d_{e, i j}^{2}
\end{aligned}
$$

- Replacing $d_{e, i j}^{2}$ with the dissimilarity OM, LCP, LCS ... (or its square), we get a pseudo SS.


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## Dissimilarity based analysis

- If we know how to compute a dissimilarity,
- we can apply any analysis based on dissimilarities or variances
- Clustering (agglomerative, divisive, partitioning, ...) (Kaufman and Rousseeuw, 2005)
- Principal coordinate analysis (PCO, MDS) (Gower, 1966)
- Representative sequences (Gabadinho et al., 2009b)
- ANOVA (Studer et al., 2010)
- ...

Analysing the diversity of categorical sequences
Unsupervised clustering

## Outline

## (1) Objectives

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Analysing the diversity of categorical sequences
Unsupervised clustering
Typology

## Section outline

(3) Unsupervised clustering

- Typology
- Representative sequences


## Building a typology

- To illustrate, hierarchical clustering with Ward criterion
- Data: Cohabitational trajectories,
- 1503 sequences from the 2002 biographic survey of the SHP
- alphabet of 10 states
- yearly data, from age 20 to 45 years (length 26)

Cohabitational trajectories


## Clustering from dissimilarities

- Compute the dissimilarity matrix, for instance with TraMineR om.coh <- seqdist(seqs.coh, method="OM", sm="TRATE", indel=1)
- om.dist.coh is a $1503 \times 1503$ matrix that can be passed to any clustering method that accepts a distance matrix as input
- In R, we can use the cluster library (Maechler et al., 2005) which proposes among others
- agnes() an agglomerative method
- diana() a divisive method
- pam() partitioning around medoids
- Illustration: agglomerative method with Ward
- We use the agnes() function
- clw.coh <- agnes(om.coh, diss=T, method="ward")
- and retain the partition into 5 clusters
- cutree(clw.coh, k=5)


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## Hierarchical clustering, Ward

## Dendrogram

Cohabitational trajectories

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## Rendering the clusters

- State sequences can easily be visualized
- Example: i-plot

Sequence
$1(2 P, 1)-(U, 25)$
$2(2 P, 2)-(U, 6)-(U C, 18)$
$3(2 P, 1)-(A, 5)-(U, 2)-(U C, 18)$
4 (2P, 8)-(U, 1)-(UC, 17)


- (Horizontal) stacking of rectangles, with color representing the state and length its duration.
- The vertical alignment informs about the distribution at each position


## i-plot, and order of the sequences

- When number of sequences is high, sorting sequences helps readability


Analysing the diversity of categorical sequences
Unsupervised clustering
Typology

## Typology of cohabitational state sequences i-plot, sorted with MDS[1]



Type 4: Parental Home Trajectories (7\%)


Type 2 : Conjugal Trajectories (17 \%)
 A20 A23 A26 A29 A32 A35 A38 A41 A44

Type 3 : Mixed Cohabitation Trajectories (8\%)


Type 5 : Solo Trajectories (5 \%)


$$
\begin{aligned}
& \square \mathrm{PP} \\
& \square \mathrm{PP} \\
& \square \mathrm{PP} \\
& \square \mathrm{~A} \\
& \square \mathrm{U} \\
& \square \mathrm{UC} \\
& \square \mathrm{UN} \\
& \square \mathrm{C} \\
& \square \mathrm{~F} \\
& \square \mathrm{O}
\end{aligned}
$$

## Typology of cohabitational state sequences

## Discrepancies

- Discrepancy (pseudo variance) $\frac{1}{2 n^{2}} \sum_{i} \sum_{j} d(i, j)$

|  | Count | Percent | Discrepancy |
| ---: | ---: | ---: | ---: |
| Parental | 938 | 62.4 | 7.819 |
| Conjugal | 261 | 17.4 | 8.209 |
| Mixed | 121 | 8.1 | 19.842 |
| Parental Home | 108 | 7.2 | 3.002 |
| Solo | 75 | 5.0 | 9.185 |
| Total | 1503 | 100.0 | 15.526 |

Analysing the diversity of categorical sequences
Unsupervised clustering
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## Typology of cohabitational state sequences d-plot, transversal distributions



Type 4: Parental Home Trajectories (7\%)


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Unsupervised clustering

## Typology

## Typology of cohabitational state sequences

## Ht-plot, transversal entropies

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Unsupervised clustering
Typology

## Typology of cohabitational state sequences mt-plot, mean time in each state

Type 1 : Parental Trajectories (62 \%)


Type 2 : Conjugal Trajectories (17\%)


Type 3 : Mixed Cohabitation Trajectories (8\%)


Type 5 : Solo Trajectories (5\%)


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## Analysing the diversity of categorical sequences

Unsupervised clustering
Typology

## MDS: Cloud of points




Analysing the diversity of categorical sequences
Unsupervised clustering
Typology

## Typology of occupational state sequences i-plot



Type 4: At Home Trajectories (9.5 \%)


Type 2: Mixed Occupational Trajectories (22 \%)
 A20 A23 A26 A29 A32 A35 A38 A41 A44

Type 3: Return Trajectories (11 \%)
 A20 A23 A26 A29 A32 A35 A38 A41 A44

Analysing the diversity of categorical sequences
Unsupervised clustering
Typology

## Typology of cohabitational state sequences d-plot



Type 2: Mixed Occupational Trajectories (22 \%)


Type 5: Part Time Trajectories (5.5 \%)



Type 3: Return Trajectories (11 \%)

Unsupervised clustering
Representative sequences

## Section outline

(3) Unsupervised clustering

- Typology
- Representative sequences


## Representative sequences

- Aim: summarize a set of sequences
- Find a small set of sequences, such that
- non redundant
- cover a minimal percentage of the set
- Redundance and coverage defined in terms of neighborhood
- $x$ and $y$ non redundant if $d(x, y)>\delta_{\text {tsim }}$
- coverage: \% of sequences that have at least one representative $r$ in their neighborhood $\left(d(x, r)<\delta_{\text {tsim }}\right)$


## Representative sequences

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## Representative sequences <br> Heuristic

(1) Sort sequences according to a representativeness criterion

- density number of sequences in its neighborhood
- centrality sum of distances to all other sequences
- others: frequency, mean of its state frequencies, likelihood, ...
(2) Suppress redundancy
- Compute coverage of the sequence with highest score
- Then, for the next ones
- drop out if redundant with sequences already retained
- else, compute coverage of the new set of representatives
- Stop when the wanted coverage is reached.

Analysing the diversity of categorical sequences
Unsupervised clustering
Representative sequences

## Representative sequences: Example

## Cohabitational trajectories ( $\mathrm{tsim}=.2$, trep=.6)



- With representative sequences, we miss small groups (Tanguy)


## Representative sequences：Example

## Cohabitational trajectories（ $\mathrm{tsim}=.2$ ，trep＝．6）


－With representative sequences，we miss small groups（Tanguy）

## Analysing the diversity of categorical sequences

## Unsupervised clustering

Representative sequences

## Representative sequences by cluster

## Cohabitational trajectories ( $\mathrm{tsim}=.1$, trep $=.4$ )

Type 1 : Parental Trajectories (62 \%)
(A) Discrepancy (mean dist. to center)
(B) Mean dist. to representative seq.


Type 4: Parental Home Trajectories (7\%)
(A) Discrepancy (mean dist. to center)
(B) Mean dist. to representative seq.


Type 2 : Conjugal Trajectories (17 \%)
(A) Discrepancy (mean dist. to center)
(B) Mean dist. to representative seq.


Type 3 : Mixed Cohabitation Trajectories (8\%)
(A) Discrepancy (mean dist. to center)
(B) Mean dist. to representative seq.
$\left.\begin{array}{ll}\widehat{\text { N }} & \mathrm{A} \\ \stackrel{I I}{\text { II }} & \mathrm{B}\end{array}\right]$


Type 5 : Solo Trajectories (5 \%)
(A) Discrepancy (mean dist. to center) (B) Mean dist. to representative seq.

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## Analysing the diversity of categorical sequences

## Unsupervised clustering

Representative sequences

## Representative sequences by cluster

## Occupational trajectories ( $\mathrm{tsim}=.1$, trep $=.4$ )

Type 1: Full Time Trajectories (52 \%)
(A) Discrepancy (mean dist. to center (B) Mean dist. to representative seq.


Type 4: At Home Trajectories (9.5 \%)
(A) Discrepancy (mean dist. to center) (B) Mean dist. to representative seq.


Type 2: Mixed Occupational Trajectories (22 \%)
(A) Discrepancy (mean dist. to center)
(B) Mean dist. to representative seq.


Type 3: Return Trajectories (11\%)
(A) Discrepancy (mean dist. to center) (B) Mean dist. to representative seq.
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## Outline

## (1) Objectives

(2) Pairwise dissimilarities between sequences
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ANOVA on sequential data

## Section outline

4 Supervised Classification

- ANOVA on sequential data
- Regression trees for categorical sequences


## Association between sequences and a covariate

- For supervised clustering (according the value of a covariate)
- ... must be able to measure association between sequential data and a covariate
- Since we know how to determine the dicrepancy

$$
S S=\frac{1}{n} \sum_{i=1}^{n} \sum_{j=1+1}^{n} d_{i j}
$$

- We can compute pseudo $R^{2}$ 's and pseudo $F^{\prime}$ s


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## Analysis of sequence discrepancy (Studer et al., 2009, 2010)

- ANOVA like analysis based on pairwise dissimilarities
- We decompose the SS (Sum of squares equivalent)

$$
S S_{T}=S S_{B}+S S_{W}
$$

- Here, with the formula shown earlier

$$
\begin{aligned}
S S_{T} & =\frac{1}{n} \sum_{i=1}^{n} \sum_{j=i+1}^{n} d_{i j} \\
S S_{W} & =\sum_{g}\left(\frac{1}{n_{g}} \sum_{i=1}^{n_{g}} \sum_{j=i+1}^{n_{g}} d_{i j, g}\right) \\
S S_{B} & =S S_{T}-S S_{W}
\end{aligned}
$$

## Pseudo R-square and ANOVA Table

- ANOVA table for $m$ groups

|  | Discrepancy | $d f$ | Mean Discr. | $F$ |
| :--- | :---: | :---: | :---: | :---: |
| Between | $S S_{B}$ | $d f_{B}=m-1$ | $\frac{S S_{B}}{d f_{B}}$ | $\frac{S S_{B}}{S S_{W}} \frac{d f_{W}}{d f_{B}}$ |
| Within | $S S_{W}$ | $d f_{W}=\sum_{g} n_{g}-m$ | $\frac{S S_{W}}{d f_{W}}$ |  |
| Total | $S S_{T}$ | $d f_{T}=n-1$ |  |  |

- Pseudo $R^{2}$

$$
R^{2}=\frac{S S_{B}}{S S_{T}}
$$

## Pseudo R-square and ANOVA Table

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| Total | $S S_{T}$ | $d f_{T}=n-1$ |  |  |

- Pseudo $R^{2}$

$$
R^{2}=\frac{S S_{B}}{S S_{T}}
$$

Analysing the diversity of categorical sequences
Supervised Classification
ANOVA on sequential data

## Pseudo F

- Pseudo F

$$
F=\frac{S S_{B} /(m-1)}{S S_{W} /(n-m)}
$$

- Normality is not defendable in this setting.
- $F$ cannot be compared with an $F$ distribution.
- The significance is assessed through a permutation test
- Permutation test: iteratively randomly reassign each covariate profile to one of the observed sequence and recompute the $F$.
- Empirical distribution of $F$ under independence.


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## Analysis of sequence discrepancy

Cohabitational trajectories with birth cohort

|  | SS | df | MSE |
| ---: | ---: | ---: | ---: |
| Exp | 186.63 | 2 | 93.32 |
| Res | 23149.63 | 1500 | 15.43 |
| Total | 23336.26 | 1502 | 15.54 |


|  | t0 | p.value |
| ---: | ---: | ---: |
| Pseudo F | 5.057 | 0.001 |
| Pseudo Fbf | 5.851 | 0.001 |
| Pseudo R2 | 0.007 | 0.001 |
| Bartlett | 8.731 | 0.001 |
| Levene | 14.122 | 0.001 |

## Empirical F distribution

Cohabitational trajectories with birth cohort


Cohabitational trajectories with month of birth


## Simple ANOVA, cohabitational trajectories

Simple ANOVA, cohabitational trajectories

|  | categories | $R^{2}$ | $F$ | Sig. |
| :--- | :---: | :---: | :---: | :---: |
| Birth Cohort | 3 | 0.008 | 6.0 | 0.001 |
| Sex | 2 | 0.004 | 6.6 | 0.001 |
| Education Level | 4 | 0.007 | 3.3 | 0.001 |
| Birth Month | 12 | 0.009 | 1.1 | 0.272 |

## Simple ANOVA, occupational trajectories

Simple ANOVA, occupational trajectories

|  | categories | $R^{2}$ | $F$ | Sig. |
| :--- | :---: | :---: | :---: | :---: |
| Birth Cohort | 3 | 0.007 | 5.1 | 0.001 |
| Sex | 2 | 0.183 | 336.6 | 0.001 |
| Education Level | 4 | 0.065 | 34.8 | 0.001 |

## Homogeneity of within group discrepancy

- Is discrepancy the same in all groups?
- Contribution to inertia $d_{x \tilde{g}}=\frac{1}{n}\left(\sum_{i} d_{x i}-S S\right)$
- Letting $z_{i}$ be the dissimilarity between sequence $i$ and its group center
- Levene: $F$ test (ANOVA) on the $z_{i}$ 's


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## Homogeneity of within group discrepancy

Levene test, Cohabitational Trajectories

|  | categories | $L$ | Sig. |
| :--- | :---: | :---: | :---: |
| Birth Cohort | 3 | 0.1 | 0.915 |
| Sex | 2 | 9.2 | 0.002 |
| Education Level | 4 | 2.4 | 0.063 |
| Birth Month | 12 | 1.3 | 0.238 |

# Homogeneity of within group discrepancy 

Levene test, Occupational Trajectories

|  | categories | $L$ | Sig. |
| :--- | :---: | :---: | :---: |
| Birth Cohort | 3 | 14.1 | 0.001 |
| Sex | 2 | 912.8 | 0.001 |
| Education Level | 4 | 15.2 | 0.001 |

## Multi-factor ANOVA

- Generalization to multi-factor case voir (Studer et al., 2010)
- Here, we consider Type II effects
- Measures contribution added by each factor $v$ when we control for all the others.
- The $F$ statistic is

$$
F_{v}=\frac{\left(S S_{B_{c}}-S S_{B_{v}}\right) / p}{S S_{W_{c}} /(n-m-1)}
$$

where $S S_{B_{c}}$ and $S S_{W_{c}}$ are the explained and residual $S S$ of the full model. $S S_{B_{v}}$ the explained part of the model after deletion of $v$, and $p$ the number of indicator or contrasts used for coding variable $v$.

- Significance is again evaluated through permutation tests.


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## Multi-factor, derivation of the results

- Consider the linear model $\mathbf{Y}=\mathbf{X B}$
- Its 'Hat' matrix $\mathbf{H}=\mathbf{X}\left(\mathbf{X}^{\prime} \mathbf{X}\right)^{-1} \mathbf{X}^{\prime}$, such that $\hat{\mathbf{Y}}=\mathbf{H Y}$
- Gower's matrix $\mathbf{G}=-\frac{1}{2}\left(\mathbf{I}-\frac{1}{n} \mathbf{1 1 ^ { \prime }}\right) \mathbf{D}\left(\mathbf{I}-\frac{1}{n} \mathbf{1 1 ^ { \prime }}\right)$ with $D$ matrix of squared Euclidean distances
- We have

$$
\begin{aligned}
S S_{T} & =\operatorname{tr}(\mathbf{G}) \\
S S_{B} & =\operatorname{tr}(\mathbf{H G}) \\
S S_{W} & =\operatorname{tr}((\mathbf{I}-\mathbf{H}) \mathbf{G})
\end{aligned}
$$

- Generalization by substituting $D$ with the matrix of dissimilarities.


## Multi-factor analysis

sex, birth cohort, level of education

Multi-factor analysis, Occupational trajectories

| Variable | PseudoF | PseudoR2 | P_value |
| :--- | ---: | ---: | ---: |
| sex | 497.039 | 0.226 | 0.0000 |
| cohort3b | 5.281 | 0.005 | 0.0010 |
| edu_lev | 34.353 | 0.047 | 0.0000 |
| Total | 116.800 | 0.319 | 0.0000 |

## Evolution of the differences

- How do differences vary over time?
- At which age do trajectories most differ between cohorts?
- Compute $R^{2}$ on small sliding windows (length 2)
- $=>$ Series of $R^{2}$, and we plot their evolution
- Likewise we can plot the series of
- the total residual discrepancies (SS $W$ )
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## Series of R-squares

## $R^{2}$, Occupational Trajectories, Birth cohort



## Series of residual discrepancies

Birth cohort


## Supervised Classification

## ANOVA on sequential data

## Series of residual discrepancies

## Birth cohort

Occupational Trajectories



Analysing the diversity of categorical sequences
Supervised Classification
Regression trees for categorical sequences

## Section outline

4 Supervised Classification

- ANOVA on sequential data
- Regression trees for categorical sequences


## Tree structured analysis of sequence data

- Aim: Find out most important predictors and their interactions.
- Iteratively segment cases using covariate values
- Form as homogeneous groups.
- at each step, select covariate and split that generates the highest $R^{2}$.
- Split significance tested with permutation F.
- Stop when the selected split is not significant.


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Analysing the diversity of categorical sequences

## Supervised Classification

Regression trees for categorical sequences

## Growing the tree

```
|-- Root (n: 1503 disc: 15)
    |-> sex 0.13
        |-- [ ] (n: 752 disc: 9.4)
        |-> edu_lev 0.072
        |-- [ ] (n: 157 disc: 8.7)[(ED,6)-(FT,20)] *
        |-- [ ] (n: 595 disc: 8.7)[(FT,26)] *
        |-- [ ] (n: 751 disc: 16)
            |-> edu_lev 0.017
            |-- [ ] (n: 632 disc: 16)
            |-> edu_lev 0.009
            |-- [ ] (n: 116 disc: 17)[(FT,9)-(AH,17)] *
            |-- [ ] (n: 516 disc: 15)
            |-> cohort3b 0.0068
            |-- [ ] (n: 280 disc: 15)[(FT,10)-(AH,10)-(PT,6)] *
            |-- [ ] (n: 236 disc: 15)[(FT,12)-(AH,14)] *
            |-- [ ] (n: 119 disc: 16)[(ED,1)-(FT,14)-(PT,11)] *
```

Analysing the diversity of categorical sequences
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## Rendering the tree

## Occupational trajectories



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Regression trees for categorical sequences

## Quality of the tree

Occupational trajectories

ANOVA for the leaves of the tree,

|  | t0 | p.value |
| ---: | ---: | ---: |
| Pseudo F | 62.49 | 0.000 |
| Pseudo Fbf | 55.66 | 0.000 |
| Pseudo R2 | 0.17 | 0.000 |
| Bartlett | 60.60 | 0.000 |
| Levene | 43.66 | 0.000 |

Analysing the diversity of categorical sequences
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## Rendering the tree

Cohabitational trajectories


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## Rendering the tree

Cohabitational trajectories


Regression trees for categorical sequences

## Quality of the tree

Cohabitational trajectories

ANOVA for leaves of the tree,

|  | t0 | p.value |
| ---: | ---: | ---: |
| Pseudo F | 5.74 | 0.000 |
| Pseudo Fbf | 5.62 | 0.000 |
| Pseudo R2 | 0.01 | 0.000 |
| Bartlett | 0.96 | 0.049 |
| Levene | 2.73 | 0.041 |

## Outline

## （1）Objectives

（2）Pairwise dissimilarities between sequences
（3）Unsupervised clustering

4 Supervised Classification
（5）Conclusion

## Conclusion 1: About sequence analysis

- Analyse trajectories until 45 years $=>$ ignore recent generations
- Most recent birth year is 1957 (2002 - 45)
- Issues:
- Granularity: year, month, day,
- State definition: should we distinguish \{separated, divorced, widowed\} or consider a single state? works by Raffaella Piccaretta


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## Conclusion 2: Missing data and weights

- Missing data in sequences
- TraMineR allows for differentiated handling of left, right and in-between missing values
- consider 'missing' as a specific state
- drop out (left shift of subsequent elements)
- impute, but how?
- Weighting cases
- Account for them in rendering of sequences (weighted transversal characteristics)
- Implemented solutions for ANOVA and permutation test
- Not relevant for dissimilarities and longitudinal characteristics


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## Conclusion 3: Extending the analysis

- Since TraMineR is an R library, its outcome can easily be combined in a same script with any other $R$ process
- We have seen: cluster analysis, MDS, ...
- In Widmer and Ritschard (2009),
- Relationship between occupational and cohabitational trajectories by regressing longitudinal entropies of each of them on occupational and cohabitational types while controlling for birth cohort and sex.
- Studied also cluster membership with logistic regressions.


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## Conclusion 4: Application to other kind of data

- Discrepancy based analysis
- ... applies to any data that can be characterized by their pairwise dissimilarities.
- Only aspect specific to state sequences: their visual rendering.


## Conclusion 4: About TraMineR

- TraMineR is a unique toolbox for discrete sequence anylsis
- Can do much more than shown in this presentation,
- handling of sequence data
- conversion between states and events
- multi-channel dissimilarity for parallel sequences
- frequent and discriminant sub-sequences
- extracting association rules between sub-sequences
- ...
- ... and, like R, available for free on the CRAN http://cran.r-project.org
- See also package's web site http://mephisto.unige.ch/traminer


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Conclusion

## Thank You!

## References I

Gabadinho, A., G. Ritschard, M. Studer, and N. S. Müller (2009a). Mining sequence data in R with the TraMineR package: A user's guide. Technical report, Department of Econometrics and Laboratory of Demography, University of Geneva, Geneva.

Gabadinho, A., G. Ritschard, M. Studer, and N. S. Müller (2009b). Summarizing sets of categorical sequences. In International Conference on Knowledge Discovery and Information Retrieval, Madeira, 6-8 October, 2009, pp. 62-69. INSTICC. (Received the Best Paper Award).
Gower, J. C. (1966). Some distance properties of latent root and vector methods used in multivariate analysis. Biometrika 53(3/4), 325-338.
Kaufman, L. and P. J. Rousseeuw (2005). Finding Groups in Data. Hoboken: Wiley.
Lesnard, L. (2006). Optimal matching and social sciences. Série des Documents de Travail du CREST 2006-01, Institut National de la Statistique et des Etudes Economiques, Paris.
Levenshtein, V. (1966). Binary codes capable of correcting deletions, insertions, and reversals. Soviet Physics Doklady 10, 707-710.

## References II

Maechler, M., P. Rousseeuw, A. Struyf, and M. Hubert (2005). Package 'cluster': Cluster analysis basics and extensions. Refence manual, R-project, CRAN.
Ritschard, G., A. Gabadinho, N. S. Müller, and M. Studer (2008). Mining event histories: A social science perspective. International Journal of Data Mining, Modelling and Management 1(1), 68-90.
Studer, M., G. Ritschard, A. Gabadinho, et N. S. Müller (2009). Analyse de dissimilarités par arbre d'induction. Revue des nouvelles technologies de l'information RNTI E-15, 7-18.

Studer, M., G. Ritschard, A. Gabadinho, et N. S. Müller (2010). Discrepancy analysis of complex objects using dissimilarities. In F. Guillet, G. Ritschard, D. A. Zighed, et H. Briand (Eds.), Advances in Knowledge Discovery and Management, Volume 292 of Studies in Computational Intelligence, pp. 3-19. Berlin: Springer.
Widmer, E. and G. Ritschard (2009). The de-standardization of the life course: Are men and women equal? Advances in Life Course Research 14(1-2), 28-39.

