Computing and using the deviance with classification trees

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Outline

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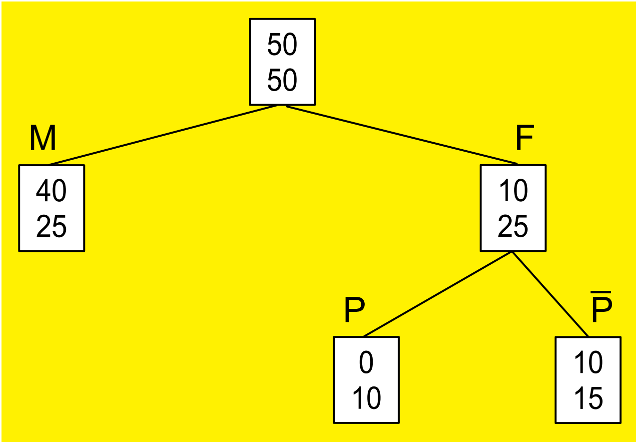
1 Introduction

- About classification trees
- Descriptive non classificatory usages
- Measuring the quality of the tree (with the deviance)
- Computational issues

Principle of tree induction

Goal: Find a partition of data such that the distribution of the outcome variable differs as much as possible from one leaf to the other.

How: Proceeds by successively splitting nodes.



- Starting with root node, seek attribute that generates the best split according to a given criterion.
- Repeat operation at each new node until some stopping criterion, a minimal node size for instance, is met.

Main algorithms:

CHAID (Kass, 1980), significance of Chi-Squares
CART (Breiman et al., 1984), Gini index, binary trees
C4.5 (Quinlan, 1993), gain ratio

2 Motivation

In social sciences, induced trees are most often used for descriptive (non classificatory) aims.

Examples:

- Mobility trees between social statuses of sons, fathers and grandfathers (data from act of marriage in the 19th century Geneva)
  (Ritschard and Oris, 2005)
  Goal: How do the statuses of the father and grandfather affect the chances of the groom to be in a lower, medium or high position?

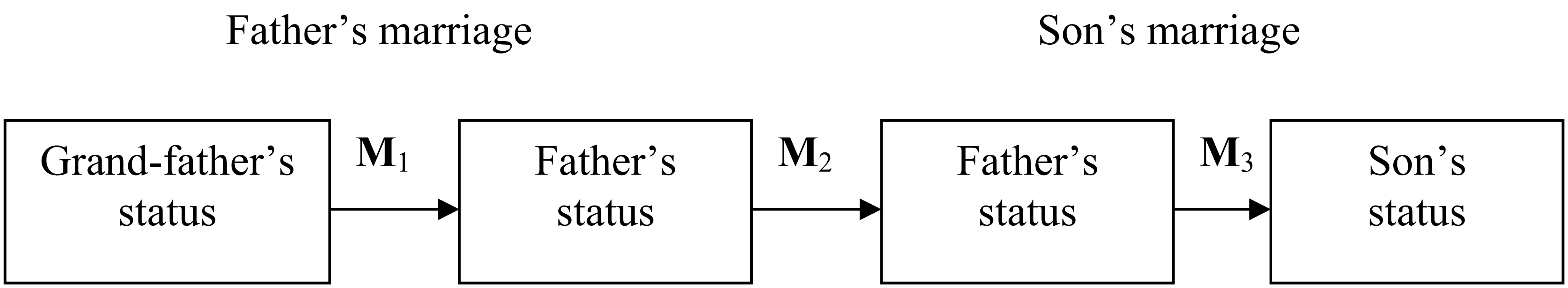
- Determinants of women's labor participation (Swiss census data)
  (Losa et al., 2006)
  Goal: How do age, number of children, education, etc. affect the chances of the woman to work at full time, long part time, short part time or not to work at all?

Mobility tree

Statuses defined from profession mentioned in marriage acts.

Acts for all men having a name beginning with a "B".

For 572 cases, was possible to match with data from father's marriage
⇒ social mobility over 3 generations

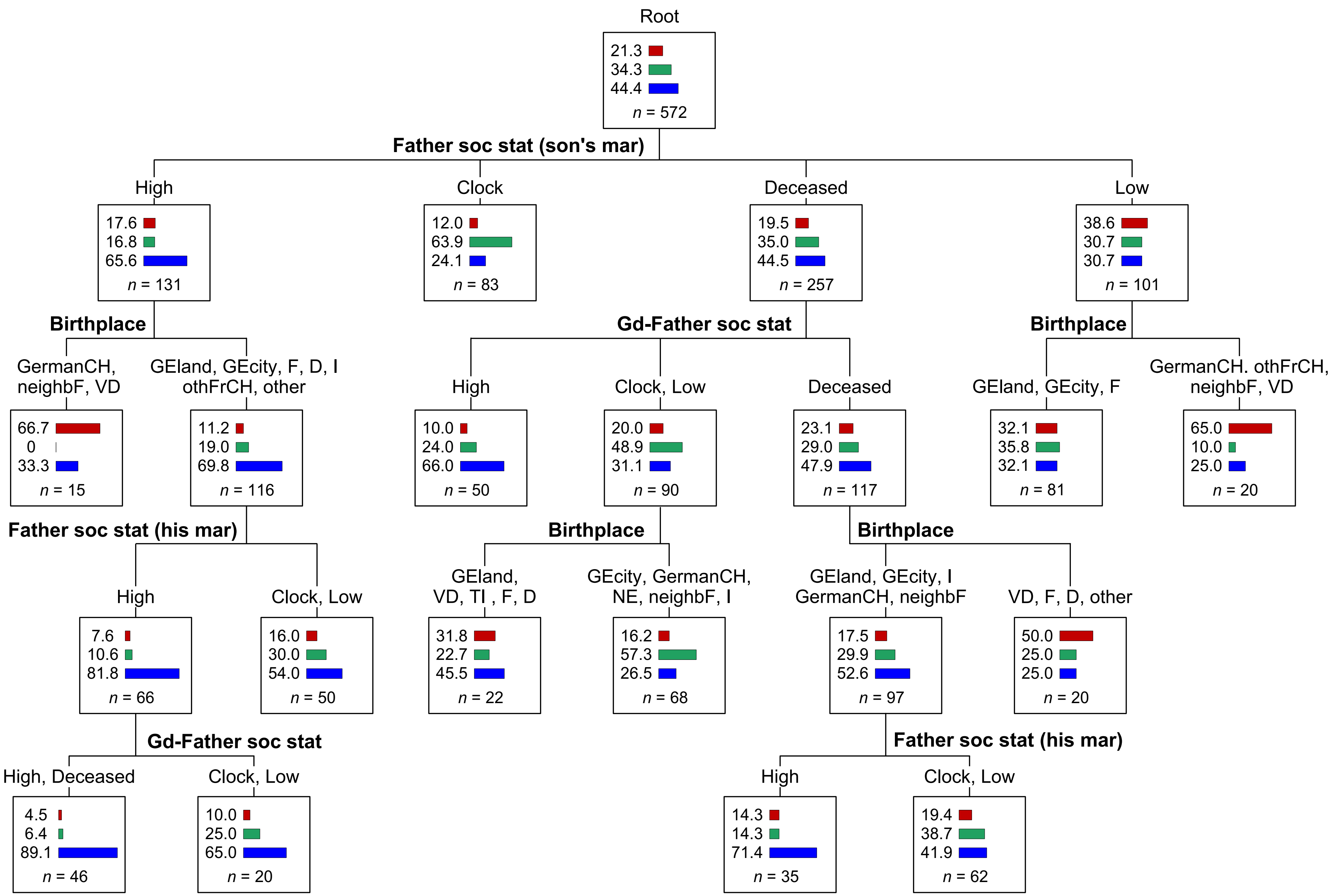


Groom's status (3 values) is response variable.

Predictors are birthplace and statuses of father and grandfather.

Method: CHAID (sig 5%, minimal child node size = 15, parent node = 30)

Mobility tree. Son's Status: Low (workers and craftmen), Clock Maker, High



Validating Tree in a Non-classificatory Setting

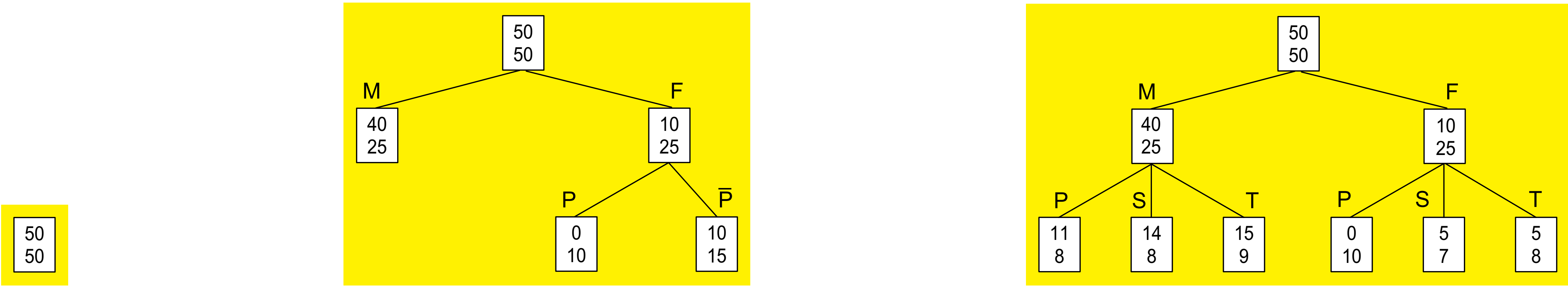
- Trees are usually validated with the classification error rate (on test data or through cross-validation)
- Claim: Classification error rate not suited for non classificatory purposes

  Example: Split into two groups with distribution

$$\begin{bmatrix} 10\% \\ 90\% \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 45\% \\ 55\% \end{bmatrix}$$

  - Distributions clearly different (valuable knowledge)
  - Split does not improve the error rate (assuming majority rule).
- Our suggestion (Ritschard and Zighed, 2003): Use the deviance for measuring the descriptive power of a tree.

3 Deviance for Trees



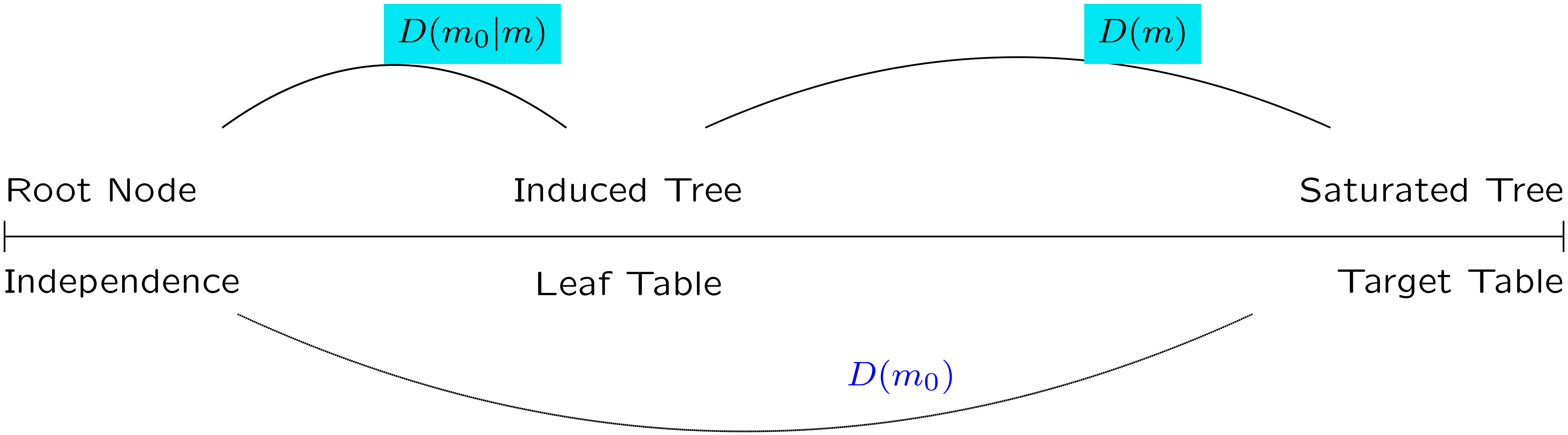
50
50

↔

40	0	10
25	10	15

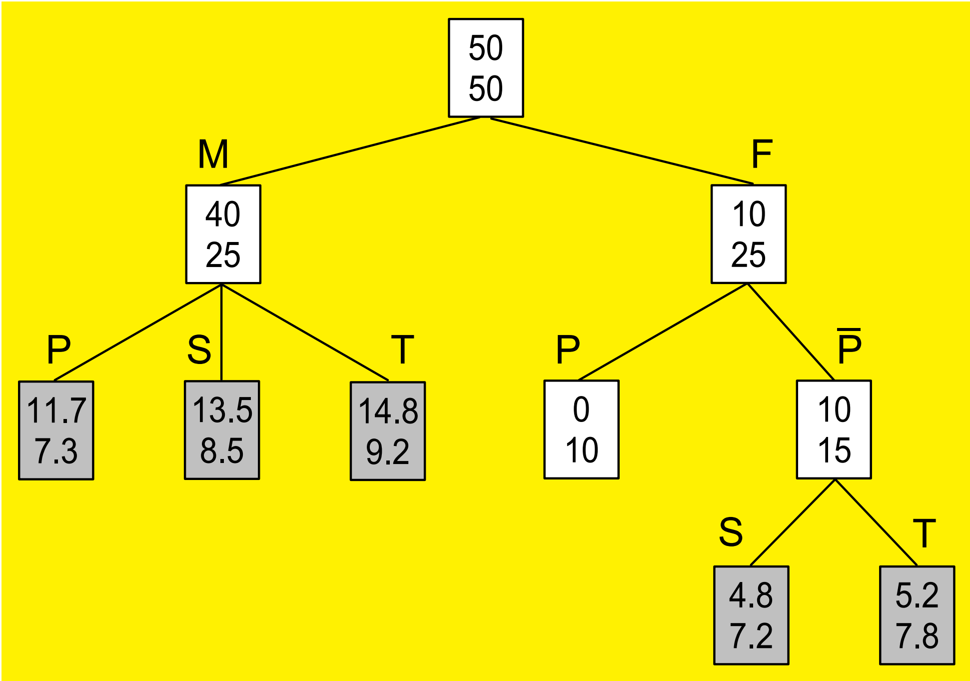
↔

11	14	15	0	5	5
8	8	9	10	7	8

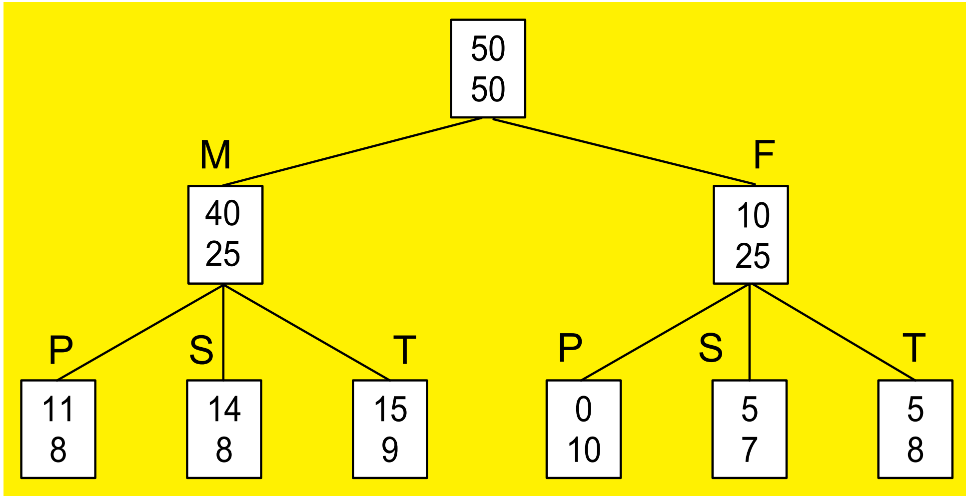


Target and Predicted Tables

Predicted Table $\hat{T}$



Target Table T



$$\hat{T} = \begin{array}{|cccccc|} 11.7 & 13.5 & 14.8 & 0 & 4.8 & 5.2 \\ 7.3 & 8.5 & 9.2 & 10 & 7.2 & 7.8 \end{array}$$

$$T = \begin{array}{|cccccc|} 11 & 14 & 15 & 0 & 5 & 5 \\ 8 & 8 & 9 & 10 & 7 & 8 \end{array}$$

Deviance: Formal Definition

$T = (n_{ij})$ $r \times c$ target table:

r rows = categories of the outcome variable

c columns = different profiles in terms of the predictors

$\hat{T} = (\hat{n}_{ij})$ $r \times c$ table predicted from the tree

Total of each column (profile) distributed according to the distribution in the leaf to which the profile belongs

$$D(m) = -2 \sum_{i=1}^{r} \sum_{j=1}^{c} n_{ij} \ln\left(\frac{\hat{n}_{ij}}{n_{ij}}\right)$$

Under regularity conditions (Bishop et al., 1975):

- $D(m) \sim \chi^2$ with $d = (r-1)(c-q)$ degrees of freedom (see Ritschard and Zighed, 2003)
- $D(m_2|m_1) = D(m_2) - D(m_1) \sim \chi^2$ with $d_2 - d_1$ degrees of freedom if m_2 restricted version of m_1

Deviance based indicators

BIC: deviance penalized for complexity (nbr of parameters)

BIC$= D(m) - d\,\ln(n) +$constant

pseudo R^2 McFadden $R^2 = 1 - D(m)/D(m_0)$,

Nagelkerke $R^2 = \dfrac{1 - \exp\{\frac{2}{n}\big(D(m_0) - D(m)\big)\}}{1 - \exp\{\frac{2}{n} D(m_0)\}}$

Theil's u (proportion of reduction of Shannon's entropy)

$$u = \frac{D(m_0|m)}{-2\sum_i n_{i.} \ln(n_{i.}/n)}$$

Evolves quadratically between independence and full association
$\Rightarrow \sqrt{u}$ represents position between the 2 extremes.

4 Outcome for the mobility tree example

- Error rate: 42.4%, (55.6% at the root node; 10 folds CV: 51.4% error)
- Goodness of fit

Tree m	$D(m)$	df	sig	BIC	AIC	Theil $\sqrt{u}$
Indep	482.3	324	0.000	2319.6	812.3	0
Level 1	408.2	318	0.000	1493.9	750.2	0.25
Level 2	356.0	310	0.037	1492.5	714.0	0.32
Level 3	327.6	304	0.168	1502.2	697.6	0.36
Fitted	312.5	300	0.298	1512.5	690.5	0.37
Saturated	0	0	1	3104.7	978.0	0.63

Between level deviance improvement

D(row model)$-D$(column model)

	Level 1	Level 2	Level 3	Fitted	Saturated
Indep	74.1^{***}	126.3^{***}	154.7^{***}	169.8^{***}	482.3^{***}
	(6 *df*)	(14 *df*)	(20 *df*)	(24 *df*)	(324 *df*)
Level 1		52.2^{***}	80.6^{***}	95.7^{***}	408.2^{***}
		(8 *df*)	(14 *df*)	(18 *df*)	(318 *df*)
Level 2			28.4^{***}	43.5^{***}	356^{**}
			(6 *df*)	(10 *df*)	(310 *df*)
Level 3				15.1^{***}	327.6
				(4 *df*)	(304 *df*)
Fitted					312.5
					(300 *df*)

*** significant at 1%, ** at 5%, * at 10%

Between level BIC variation

BIC(row model)−BIC(column model)

	Level 1	Level 2	Level 3	Fitted	Saturated
Indep	825.7	827.1	817.4	807.1	-785.1
Level 1	0	1.4	-8.3	-18.6	-1610.8
Level 2		0	-9.7	-20	-1612.2
Level 3			0	-10.3	-1602.5
Fitted				0	-1592.2

From the BIC standpoint, Level 1 and Level 2 models look the most interesting.

5 Computational Issues

1. Softwares for growing trees do not provide
   - the deviance
   - nor easily usable information for computing the target and predicted tables

   Solution: look at LR statistics for cross tables.

2. Number of possible profiles (columns) may become excessively large.

   May be as large as $$\prod_{\nu=1}^{V} c_\nu$$

   with c_ν the number of values of the v-th predictor

   Solution: partial deviance (distance to a smaller arbitrary target table.)

Deviance and Likelihood Ratio Chi-squares

$D(m_0|m)$ = LR Chi-square statistic for testing independence on Leaf Table (crosstabulation of response variable with leaf variable).

$D(m_0)$ = LR Chi-square statistic for testing independence on Target Table (crosstabulation of response variable with profile variable).

These statistics can easily be computed with most statistical package (SPSS, SAS, ...)

Deviance of Tree m is just their difference

$$D(m) = D(m_0) - D(m_0|m)$$

Need just to retrieve for each case:

- leaf number
- profile number

Partial deviance $D(m|m_{T^*})$

Arbitrary $r \times c^*$ target table T^*

defined from the c^* profiles in terms of the mere predictors and value groupings retained by the induced tree.

Due to arbitrariness of T^*

- Deviance $D(m_{T^*})$ is no longer distance to true target.
- Pseudo R^2's based on $D(m_{T^*})$ are irrelevant.

Differences of deviances between nested trees are independent of the target. For example:

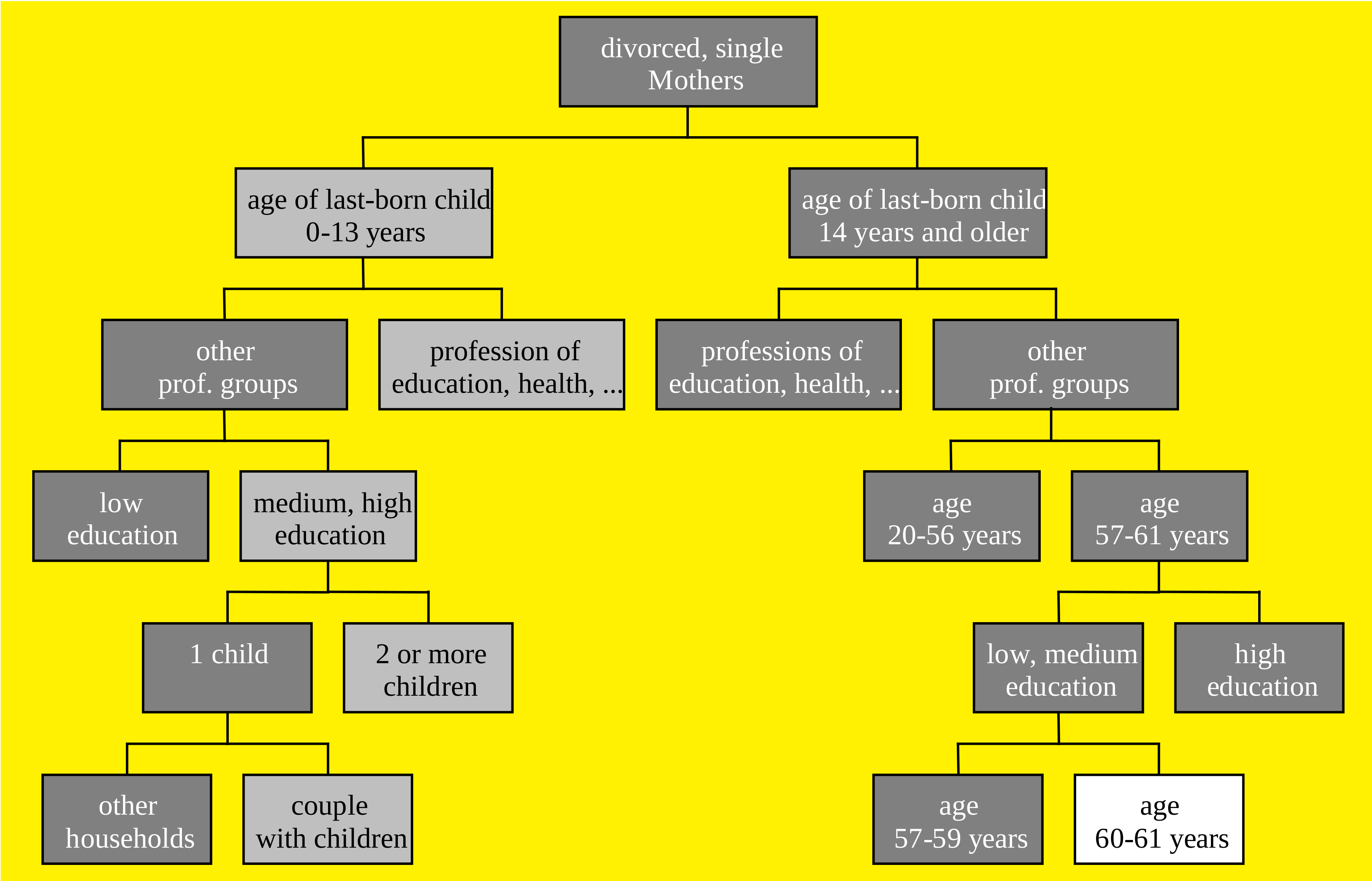
$$D(m_0|m) = D(m_0) - D(m) \quad = \quad D(m_0|m_{T^*}) - D(m|m_{T^*})$$

measures the gain over the root node (as the classical Chi-square used with logistic regression).

BIC and $\sqrt{u}$ can still be used.

6 Women's labour participation example

Tree for participation of divorced or single mothers, French speaking region.



Quality of the trees

	q	c^*	p	n	$D(m_0\|m)$	d	sig.
CHI	12	263	299	5770	822.2	33	.00
CHF	10	644	674	35239	4293.3	27	.00
CHG	11	684	717	99641	16258.6	30	.00

	ΔBIC(m_0, m)	ΔBIC(m_{T^*}, m)	u Theil	$\sqrt{u}$
CHI	536.4	3235.7	.056	.237
CHF	4010.7	4160.0	.052	.227
CHG	15913.3	-17504.3	.064	.253

7 Conclusion

Summary:

- Deviance may be used with trees.
- Deviance and differences in deviances useful for evaluating the descriptive power of trees.
- Deviance based measures, such as BIC and Theil's u, also useful.
- Computation issues: solutions exist.

Further issues for descriptive trees:

- Using BIC as tree growing criterion.
- Evaluating the stability of induced trees (Dannegger, 2000).

THANK YOU

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